Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

Theoretical model for layer reorientation in smectic A* liquid crystals subject to an oblique magnetic field

Tomas Carlsson[a]
[a] Physics Department, Division of Microelectronics and Nanosciences, Chalmers University of Technology, S-412 96 Goteborg, Sweden,

Online publication date: 06 August 2010

To cite this Article Carlsson, Tomas(2000) 'Theoretical model for layer reorientation in smectic A* liquid crystals subject to an oblique magnetic field', Liquid Crystals, 27: 7, 957 — 963
To link to this Article: DOI: 10.1080/02678290050043914
URL: http://dx.doi.org/10.1080/02678290050043914

Liquid Crystals, 2000, Vol. 27, No. 7, 957–963

Theoretical model for layer reorientation in smectic A* liquid crystals subject to an oblique magnetic field

TOMAS CARLSSON*

Physics Department, Division of Microelectronics and Nanosciences, Chalmers University of Technology, S-412 96 Göteborg, Sweden

(*Received 26 October 1999; accepted 8 February 2000*)

Recently, Carlsson and Osipov reported on observations of the rotation of smectic layers under certain experimental conditions, and a dynamic theory describing such rotations was presented [Carlsson, T., and Osipov, M. A., 1999, *Phys. Rev. E*, **60,** 5619]. In the present work, the rotational motion of the smectic layers of a SmA* liquid crystal, over which an oblique magnetic field has been applied, is studied theoretically by the aid of this theory. The role of the interaction between the smectic layers and the substrates is discussed and it is shown that if this interaction is small enough, the smectic layers orient themselves in such a way that the layer normal is almost parallel to the magnetic field. The relevant material parameters governing the dynamical response of the induced tilt and the layer normal are identified, and the time dependence of the response of these two dynamical variables is calculated. From this calculation, expressions for the response times of the system are given and a numerical value of the rotational viscosity of the smectic layers is estimated.

1. Introduction

When external torques are applied to smectic liquid crystals, the response of the system is in most cases analysed on the assumption that the smectic layers remain fixed. Thus, when an electric or magnetic field is applied over a chiral or non-chiral smectic A or smectic C liquid crystal, the study of the behaviour of the system is normally restricted to the study of the change of the tilt of the director (the soft mode) and/or the change of phase of the *c*-director. In this approach it is assumed that the interaction between the smectic layers and the substrates normally surrounding a liquid crystalline sample is so strong, that the smectic layers remain fixed irrespective of which external torques are applied to the system. In most experimental situations this approach seems to be justified.

The ordering of smectic C* (SmC*) liquid crystals can generally be described by specifying the tilt θ of the director with respect to the smectic layer normal and by two unit vectors. These two vectors represent the layer normal **a** and the *c*-director **c**, the latter denoting the tilt direction of the director **n** within the smectic layers. The effect of an external torque Γ^{ext} applied over a smectic liquid crystal is most easily investigated by dividing the torque into one part $\Gamma_{\parallel}^{\text{ext}}$ which is parallel to the layer normal **a** and another part $\Gamma_{\perp}^{\text{ext}}$ being confined within the smectic layers, i.e. $\Gamma^{\text{ext}} = \Gamma_{\parallel}^{\text{ext}}\,\mathbf{a} + \Gamma_{\perp}^{\text{ext}}\,\mathbf{d}$, where **d** represents some arbitrary unit vector parallel to the smectic layers. The torque $\Gamma_{\parallel}^{\text{ext}}$ acts to rotate the director around the smectic cone at constant tilt, while $\Gamma_{\perp}^{\text{ext}}$ tends to rotate the director in such a way that the tilt changes [1]. Changing the tilt from its equilibrium value however is normally associated with large energy changes, except when the system is very close to the SmC*–SmA* phase transition temperature T_c [2]. Because of this stiffness of the tilt, the consequence of the torque $\Gamma_{\perp}^{\text{ext}}$ is, apart from changing the tilt to a small extent, also to rotate the entire smectic layers [1].

Because the substrates which normally surround a liquid crystalline sample act to stabilize the smectic layers, these are most often observed to remain fixed in space and time even if an external torque $\Gamma_{\perp}^{\text{ext}}$ is present over the system. Mathematically, this can be expressed by introducing a counter-torque Γ^{c} which balances Γ^{ext} and has its origin in the interaction between the smectic layers and the substrates. However, recent experimental observations of macroscopic layer rotations in both the SmA* and SmC* phases have been reported in the literature [3–9]. Most of these reports concern the response of chiral smectics where asymmetrical a.c. electric fields have been applied over the system [3–8]. In one experiment [9] it is observed how the smectic layers reorient when a strong magnetic field is applied at an oblique angle to the smectic layers. In all these experiments it is obvious that the surface treatment of the liquid crystalline cell does not impose on the smectic

*Author for correspondence.

Liquid Crystals ISSN 0267-8292 print/ISSN 1366-5855 online © 2000 Taylor & Francis Ltd
http://www.tandf.co.uk/journals

layers anchoring conditions from the substrates strong enough to prevent the layers from starting to rotate.

In order to describe the rotation of the smectic layers mathematically one needs a model which couples the three macroscopic variables specifying the state of the system. These three are the tilt of the director, θ, the smectic layer normal **a** and the *c*-director, **c**. Until recently, existing models describing the dynamics of smectics either assume the tilt to be constant [1] or the smectic layers to remain fixed [10] irrespective of what torques are exerted on the director. Recently, however, a model which couples all the three macroscopic variables θ, **a** and **c** has been developed. Carlsson and Osipov [11] have shown how, in a quantitatively correct way, to describe the rotation of the smectic layers which can be observed [3, 8] when a SmA* liquid crystal in the bookshelf geometry is subject to an asymmetric saw-tooth electric field. This model was later extended [12] to describe the corresponding phenomena observable [3–8] in the SmC* phase.

Applying a strong magnetic field at an oblique angle to the smectic layers over a SmC* liquid crystal, Zalar *et al.* [9] observed a rotational motion of the smectic layer normal. Motivated by this observation, it is the aim of the present work to employ the dynamic theory by Carlsson and Osipov [11] to give a quantitatively correct description of such a reorientation of the smectic layer normal. For simplicity, the study in the present work is restricted to the case where the magnetic field is applied to a system in the SmA* phase. The outline of the paper is as follows. In § 2 the dynamical model by Carlsson and Osipov [11] is reviewed. This model demonstrates how the coupling between the rotation of the smectic layer normal and the change of tilt due to the influence of external torques can be expressed mathematically. The quantities necessary to describe the system studied and the notations adopted in this work are introduced in § 3. The equations governing the behaviour of the system are derived in § 4. Assuming the smectic layers to remain fixed, these equations are solved in § 5.1, while in § 5.2 this assumption is relaxed and the full set of equations is solved, demonstrating how the reorientation of the smectic layer normal due to the presence of the magnetic field can be described mathematically. Finally, in § 6 the results are discussed and the relevant material parameters controlling the response of the system are identified.

2. Summary of the dynamic theory governing layer reorientation in the smectic A* phase

In this section the dynamic equations governing the rotational motion of the smectic layer normal as derived by Carlsson and Osipov elsewhere [11] are summarized. This theory shows how the induced tilt θ and the influence of external fields couple to the smectic layer normal **a**, causing the latter to rotate unless a sufficiently large counter torque is applied to the smectic layers via the substrates. Here it is assumed that no such boundary conditions are present, i.e. that the smectic layers are free to rotate as soon as a proper torque is applied. After having derived (in § 4) the final equations governing the rotational motion of the smectic layers in the specific case studied here, it will be shown how to incorporate into the theory a finite frictional torque due to the substrates. As in [11], a system in the bookshelf geometry is studied, i.e. the smectic layers are at all times assumed to be perpendicular to the substrates. The state of the system is then specified by two macroscopic variables and their time derivatives. These two variables are the angle γ between the layer normal and some reference direction in the plane of the substrates and the tilt angle θ. The dynamic equations of the system can be derived by a general approach based on the Rayleigh dissipation function, as discussed in detail for the case of nematic liquid crystals by Vertogen and de Jeu [13]. The two dynamic equations governing the system studied here can in this formalism be written [11]

$$\frac{\partial D}{\partial \dot{\gamma}} = -\frac{\partial g}{\partial \gamma} \tag{1 a}$$

$$\frac{\partial D}{\partial \dot{\theta}} = -\frac{\partial g}{\partial \theta} \tag{1 b}$$

where $D = D(\dot{\gamma}, \dot{\theta})$ is the dissipation function and $g = g(\gamma, \theta)$ is the free energy density of the system. The free energy density g of course depends on which external torques are applied to the system, while the dissipation function D generally can be written as a quadratic form of the time derivatives $\dot{\gamma}$ and $\dot{\theta}$,

$$D = \frac{1}{2}\beta_1 \dot{\gamma}^2 + \frac{1}{2}\beta_2 \dot{\theta}^2 + \beta_{12}\dot{\gamma}\dot{\theta} \tag{2}$$

where the last term describes the dynamical coupling between the variables γ and θ. The dynamic coefficients β_i represent generalized viscosities and can be proven [11] to fulfill the following inequalities

$$\beta_1 > 0, \quad \beta_2 > 0, \quad |\beta_{12}| < (\beta_1 \beta_2)^{1/2}. \tag{3 a}$$

Although, as can be seen from the last of the inequalities (3 *a*), β_{12} can adopt both positive and negative values, experimental information indicates [3, 4, 14] that β_{12} is negative and therefore the present study is restricted to the case

$$\beta_{12} < 0. \tag{3 b}$$

By specifying the external fields applied over the system, starting from equations (1) and (2) a set of differential equations governing the dynamical behaviour of γ and θ will be derived in §4.

3. Rotating smectic layers in a magnetic field: introduction of notations and definition of coordinates

The system studied in this work is depicted in figure 1. It consists of a SmA* liquid crystal in the bookshelf geometry confined between two parallel glass plates. These are taken to be parallel to the xz plane. Without loss of generality (the SmA* phase is rotationally symmetric around the layer normal) the magnetic field **B** is assumed to be confined within the xz plane, the angle between the magnetic field and the z axis being η_o. In order to describe the layer normal **a**, which for standing layers is always confined within the xz plane, an angle γ is introduced, counting γ positive for a rotation of **a** around the positive y axis. With the present assumptions, the director will always be confined within the xz plane. To describe the tilt of the director with respect to the layer normal, it is thus sufficient to introduce one coordinate θ. Also θ is introduced in such a way that θ is positive for a rotation of the director with respect to the layer normal around the positive y axis. The angle between the magnetic field and the director is denoted ψ and can be expressed as

$$\psi = \eta_o - \gamma - \theta. \tag{4}$$

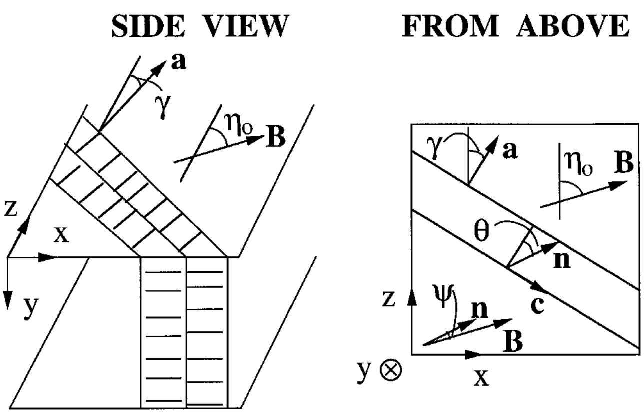

Figure 1. Definition of coordinates in the present work. Assuming that the system remains in the bookshelf geometry, two coordinates are needed in order to describe it completely. These are the angle γ between the layer normal and the z axis, and the angle θ between the director and the layer normal (the tilt). Both of these coordinates are introduced in such a way that they are positive for a rotation around the positive y axis. The magnetic field is confined within the xz plane, the angle between the field and the z axis being η_o.

With the above assumptions the layer normal **a**, the director **n** and the magnetic field **B** can be expressed as

$$a_x = \cos\gamma, \quad a_y = 0, \quad a_z = \sin\gamma \tag{5}$$

$$n_x = \cos(\gamma+\theta), \quad n_y = 0, \quad n_z = \sin(\gamma+\theta) \tag{6}$$

$$B_x = \cos\eta_o, \quad B_y = 0, \quad B_z = \sin\eta_o. \tag{7}$$

4. Derivation of the governing equations

The equations governing the dynamical behaviour of the system studied here are given by equations (1) with the dissipation function (2) and the proper free energy density g substituted into it. The liquid crystal is assumed to be either a chiral or a non-chiral smectic A material. Anyhow, for convenience we write SmA* for both these cases in the remainder of the paper. Assuming for simplicity that the magnetic anisotropy χ_a of the liquid crystal is positive, when a magnetic field is applied at an oblique angle to the smectic layers, the system will respond in the following way. Firstly a small tilt will be induced on a time scale in the millisecond regime or less. Secondly, if not too strong anchoring conditions are imposed on the smectic layers from the substrates, the layers will slowly rotate, the tilt at the same time decreasing, until the tilt is zero and the layer normal is parallel to the magnetic field. Such a rotation of the smectic layers occurs on a time scale corresponding to several seconds or even minutes [3, 4, 14].

The free energy density of the system which shall be substituted into equation (1) is the sum of the Landau energy of the SmA* phase [10] expanded to second order and the magnetic free energy density [15]

$$g = \frac{1}{2}a\theta^2 + \frac{1}{2\varepsilon}P^2 - CP\theta - \frac{\chi_a}{2\mu_o}(\mathbf{n}\ \mathbf{B})^2. \tag{8}$$

In this equation $\mu_o = 4\pi \times 10^{-7}\ \mathrm{V\,s\,A^{-1}\,m^{-1}}$ represents the permeability of free space and χ_a the magnetic anisotropy of the system, while a, ε and C are the usual Landau coefficients, where only the coefficient a is assumed to be temperature dependent,

$$a = \alpha(T - T_o). \tag{5}$$

Introducing δ as a measure of the strength of the interaction between the liquid crystal and the magnetic field,

$$\delta = \frac{\chi_a B^2}{2\mu_o} \tag{10}$$

and the free energy density of the system can be written as

$$g = \frac{1}{2}a\theta^2 + \frac{1}{2\varepsilon}P^2 - CP\theta - \delta\cos^2(\eta_o - \gamma - \theta) \tag{11}$$

where $(\eta_o - \gamma - \theta)$ is the angle between the director and the magnetic field. The equilibrium values of the tilt and polarization are derived by minimizing g with respect to θ and P,

$$\frac{\partial g}{\partial \theta} = a\theta - CP - \delta \sin 2(\eta_o - \gamma - \theta) = 0 \tag{12}$$

$$\frac{\partial g}{\partial P} = \frac{1}{\varepsilon} P - C\theta = 0. \tag{13}$$

Equation (13) implies

$$P = \varepsilon C \theta \tag{14}$$

which, when substituted into equation (12) implies

$$\frac{\partial g}{\partial \theta} = (a - \varepsilon C^2)\theta - \delta \sin 2(\eta_o - \gamma - \theta) = 0. \tag{15}$$

In the next section it is shown that, for a fixed value of γ, this equation has always a solution for a finite θ and thus the system no longer exhibits a sharp transition between the SmA* and SmC* phases, an effect that has already been discussed by Carlsson and Dahl [2]. Denoting the SmA*–SmC* phase transition temperature by T_c and introducing the shorthand notation $a_o = a - \varepsilon C^2$, in the absence of the magnetic field it can be shown [10] that

$$a_o = a - \varepsilon C^2 = \alpha(T - T_c) + \tilde{K} q_o^2 \tag{16}$$

where $\tilde{K}$ is a renormalized elastic constant and q_o is the wave vector of the pitch at T_c. Thus,

$$\frac{\partial g}{\partial \theta} = a_o \theta - \delta \sin 2(\eta_o - \gamma - \theta). \tag{17}$$

This expression is valid both for a chiral and a non-chiral system; one only has to disregard the term $\tilde{K} q_o^2$ in equation (16) in the latter case.

Allowing the smectic layer normal to rotate (i.e. γ is no longer assumed to be constant) the dynamical equations of the system are now derived from equations (1), (2), (11) and (17),

$$\beta_1 \dot{\gamma} + \beta_{12} \dot{\theta} = \delta \sin 2(\eta_o - \gamma - \theta) \tag{18 a}$$

$$\beta_2 \dot{\theta} + \beta_{12} \dot{\gamma} = -a_o \theta + \delta \sin 2(\eta_o - \gamma - \theta). \tag{18 b}$$

Equations (18) describe the dynamics of the system if no counter torque is transmitted to the smectic layers from the substrates. However, in many cases some kind of surface treatment is imposed on the glass plates surrounding the sample. Due to this, there exists a stabilizing torque Γ_s acting on the smectic layers. In order to rotate the layers, the driving torque must exceed some threshold, corresponding to the maximum possible value of the stabilizing torque, denoted by Γ_o. The stabilizing torque Γ_s has the nature of a friction torque, and adopts the value needed to balance the driving torque as long as this is not large, implying that in this case $\dot{\gamma} = 0$. However, if the driving torque exceeds a critical value, the stabilizing torque can no longer increase, but adopts its maximum value Γ_o, always opposing the rotation of the layers. Before adding this torque to the dynamical equations (18), some caution must be taken regarding the physical dimensions of the quantities studied. In equation (18 *a*), which can be regarded as the equation of motion of the layer normal, the coefficient β_1 represents a viscosity and has the unit Pa s [1], while the unit of $\dot{\gamma}$ is s^{-1}. Thus the unit of this equation is $Pa = N\,m^{-2} = Nm\,m^{-3}$ and the dimension of this equation is torque per unit volume. As the stabilizing torque Γ_s only acts via the substrates, this quantity represents a torque per unit area, i.e. $N\,m^{-1}$. Writing down the dynamical equation for *one* smectic layer, equation (18 *a*), which represents a torque per unit volume, must be multiplied by μdl, μ and d being the layer thickness and the sample thickness, respectively, and l representing the length of the smectic layer studied. The stabilizing friction torque Γ_s, which represents a torque per unit area, however, must be multiplied by $2\mu l$, the factor 2 stemming from the fact that the sample is surrounded by two glass plates. By adding Γ_s with the proper sign to equation (18 *a*), and multiplying each term with the relevant geometrical factor, one obtains the final dynamical equations of the system,

$$\beta_1 \dot{\gamma} + \beta_{12} \dot{\theta} = \delta \sin 2(\eta_o - \gamma - \theta) - \frac{2\Gamma_o}{d} \tag{19 a}$$

$$\beta_2 \dot{\theta} + \beta_{12} \dot{\gamma} = -a_o \theta + \delta \sin 2(\eta_o - \gamma - \theta). \tag{19 b}$$

These equations are valid provided that the driving torque in equation (19 *a*), $\delta \sin 2(\eta_o - \gamma - \theta) - \beta_{12}\dot{\theta}$, is larger than the maximum available stabilizing torque $2\Gamma_o/d$. If this is not the case, Γ_s will simply adopt a value large enough to achieve torque balance and equation (19 *a*) will just read $\dot{\gamma} = 0$.

5. Solution of the governing equations

The problem studied here is the following: consider a SmA or SmA* liquid crystal according to figure 1, oriented in such a way that the smectic layer normal is parallel to the z axis ($\gamma = 0$). At time $t = 0$ an oblique magnetic field is applied over the system, the angle between the field and the z axis being η_o. The dynamical response of the system is now governed by equations (19). Before solving these equations one should notice that there are two different time scales involved in the problem. The response of the tilt, being in the sub-millisecond regime, is much faster than the rotational motion of the smectic layers, which corresponds to several seconds or more. Thus, during the short time interval when the tilt adopts its equilibrium value θ_{eq} in the field, the rotation of the smectic layers is negligible. The second step of the process then corresponds to the rotational motion of the smectic layers, the tilt all the time adjusting itself to the relevant quasi-equilibrium value for a given γ. This approach is valid for the whole SmA* phase, except in a very narrow temperature interval close to T_c ($\Delta T \sim 10^{-2}$ K), where the tilt response becomes slow [2] and the small angle approximation for θ breaks down.

5.1. *The tilt response for fixed γ*

Excluding a small temperature interval close to T_c in the analysis corresponds to assuming

$$\delta \ll a_o \tag{20}$$

a relation which is easily seen to be valid provided $T - T_c > 0.01$ K [2]. Now looking for the tilt response for a fixed γ, the term $\beta_{12}\dot{\gamma}$ can be neglected in equation (19 *b*) and this equation can be approximated as

$$\beta_2\dot{\theta} = -a_o\theta + \delta \sin 2(\eta_o - \gamma - \theta) \tag{21}$$

where a_o and δ are defined by equations (16) and (10), respectively. Looking first for the equilibrium value θ_{eq} of the tilt after a long time, the equation

$$a_o\theta = \delta \sin 2(\eta_o - \gamma - \theta) \tag{22}$$

has to be solved. The features of the solution to this equation are appreciated by studying figure 2, where $(a_o/\delta)\theta$ and $\sin 2(\eta_o - \gamma - \theta)$ are plotted as functions of θ. The solution of equation (22), given by the intersection of the two graphs, is seen to vary from $\theta_{eq} = 0$ in the limit $\delta/a_o \to 0$ to $\theta_{eq} = \eta_o - \gamma$ when $\delta/a_o \to \infty$. This bahaviour is easily understood from a physical point of view, because $\delta/a_o \to 0$ corresponds to the SmA phase ($\theta_{eq} = 0$) without the magnetic field applied, while $\delta/a_o \to \infty$ corresponds to the soft limit of the system in the vicinity of T_c, $\theta_{eq} = \eta_o - \gamma$ implying that the director is parallel to the magnetic field. Assuming now δ/a_o to be small, equation (21) is expanded to first order,

$$\beta_2\dot{\theta} + a_o\theta = \delta \sin 2(\eta_o - \gamma). \tag{23}$$

From equation (23), θ_{eq} is easily derived as

$$\theta_{eq} = \frac{\delta}{a_o} \sin 2(\eta_o - \gamma) \tag{24}$$

while the time dependent response, given the initial condition $\gamma = 0$ for $t = 0$, is given by

$$\theta(t) = \theta_{eq}[1 - \exp(-a_o t/\beta)] \tag{25}$$

From equation (25) one deduces the 'magnetoclinic' response time τ_m of the system as

$$\tau_m = \frac{\beta_2}{a_o}. \tag{26}$$

In equation (26), β_2 corresponds to the soft mode rotational viscosity [10] of the SmA* phase, γ_s, and thus equations (16) and (26) imply

$$\tau_m = \frac{\gamma_s}{\alpha(T - T_c) + \tilde{K}q_o^2}. \tag{27}$$

Neglecting the renormalization $\tilde{K}q_o^2$ in the denominator of equation (27), one can estimate τ_m by inserting some typical values for α and γ_S. If these are chosen as a $\alpha \sim 5 \times 10^4$ N m^{-2} K^{-1} [2, 16] and $\gamma_S \sim 1$ Pa s [17], the estimated value of τ_m at $T - T_c = 2$ K is $\tau_m \sim 10^{-5}$ s, justifying the assumption that the two time scales involved in the problem are separated by several orders of magnitude.

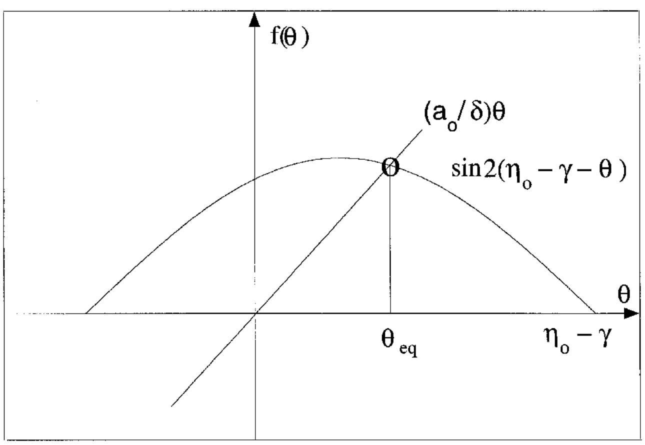

Figure 2. Graphical illustration of the solution of equation (22).

5.2. *The general rotational motion of the smectic layers*

We now proceed to study the general rotational motion of free smectic layers in an oblique magnetic field, i.e. we have to solve equations (18), which represent the special case of equations (19) assuming $\Gamma_o = 0$,

$$\beta_1\dot{\gamma} + \beta_{12}\dot{\theta} = \delta \sin 2(\eta_o - \gamma - \theta) \tag{18 a}$$

$$\beta_2\dot{\theta} + \beta_{12}\dot{\gamma} = -a_o\theta + \delta \sin 2(\eta_o - \gamma - \theta). \tag{18 b}$$

Eliminating the term $\delta \sin 2(\eta_o - \gamma - \theta)$ from these two equations one derives

$$\dot{\gamma}(\beta_1 - \beta_{12}) + \dot{\theta}(\beta_{12} - \beta_2) = a_o\theta. \tag{28}$$

Again excluding a small temperature interval close to T_c, the tilt response can be assumed to be infinitely faster than the layer rotation, and for each value of γ one can substitute the equilibrium value θ_{eq} given by equation (24) and the corresponding time derivative

$$\dot{\theta}_{eq} = -2\dot{\gamma}\frac{\delta}{a_o} \cos 2(\eta_o - \gamma) \tag{29}$$

into equation (28), thus obtaining

$$\dot{\gamma}\left[\beta_1 - \beta_{12} + 2(\beta_2 - \beta_{12})\frac{\delta}{a_o}\cos 2(\eta_o - \gamma)\right] = \delta \sin 2(\eta_o - \gamma). \quad (30)$$

The assumption (20) allows the neglect of the second term within the brackets, and the equation of motion to be solved for γ reduces into

$$\dot{\gamma}(\beta_1 - \beta_{12}) = \delta \sin 2(\eta_o - \gamma). \quad (31)$$

Applying the initial condition $\gamma = 0$ for $t = 0$, equation (31) has the solution

$$\gamma(t) = \eta_o - \arctan\{\tan(\eta_o)\exp[-2\delta t/(\beta_1 - \beta_{12})]\}. \quad (32)$$

From this equation, the response time of the smectic layer normal τ_l is seen to be

$$\tau_l = \frac{\beta_1 - \beta_{12}}{2\delta} \quad (33)$$

a quantity which is assured to be positive by the first of the inequalities (3 *a*) and the assumption (3 *b*). Thus the response of the system away from T_c can be regarded as a two-step process. First the tilt adopts its equilibrium value (24) according to the time dependence expressed by equation (25) and then, at a slower rate, the layers rotate according to equation (32), the tilt for each γ adopting its quasi-equilibrium value (24).

From equations (10) and (33) one can derive

$$\beta_1 - \beta_{12} = \frac{\chi_a B^2 \tau_l}{\mu_o} \quad (34)$$

and thus the determination of τ_l would allow for an estimation of the viscosity coefficient $\beta_1 - \beta_{12}$. Today no quantitative experiments which would allow τ_l to be evaluated are reported in the literature. However, it is known that the typical time scale for layer reorientations in electric fields is of the order of several seconds or even minutes [3, 4, 14]. Assuming here $\tau_l \sim 10$ s, $\chi_a \sim 10^{-5}$ [18] and $B \sim 5$ T, one can make the plausible estimation $\beta_1 - \beta_{12} \sim 2000$ Pa s.

6. Discussion

In this work we have discussed the dynamical response of a SmA* liquid crystal when it is subject to an oblique magnetic field. It has been shown how the recent model for layer reorientations by Carlsson and Osipov [11] can be employed to derive a set of dynamical equations governing the response of the system. The most general form of these equations is given in equations (19), consisting of two coupled differential equations, one of which can be regarded as an equation of motion for the smectic layer normal, (19 *a*), while the other represents an equation of motion for the tilt, (19 *b*). One important feature of these equations is the presence of an external counter-torque Γ_o due to the interaction between the substrates and the smectic layers. Assuming for simplicity that free smectic layers are studied, no such counter-torque is present. In this case $\Gamma_o = 0$ and the dynamical equations of the system reduce to equations (18).

As discussed in § 5, if a sufficiently small temperature interval, $\Delta T \sim 0.01$ K, in the vicinity of T_c is excluded from the analysis, the procedure of solving equations (18) is simplified. This is due to the fact that the tilt response in this case is several orders of magnitude faster than the layer response, and the tilt adopts its equilibrium value well before the layers start rotating. The layer rotation then takes place at a slower rate, the tilt all the time adopting its quasi-equilibrium value for a given orientation of the smectic layers. The time dependences of the response of the tilt and the layer normal are given by equations (25) and (32), respectively. From these one can derive expressions for the response times of the tilt τ_m, equation (26), and the layer normal τ_l, equation (33). If one wants to study the response of the system close to T_c, the mathematical problem of solving equations (18) becomes more involved, because in this case τ_m and τ_l are of the same order of magnitude and these two equations now have to be solved simultaneously. This limiting case is not studied in this work.

To achieve a good understanding of the dynamical response of the system in the most transparent way, equation (19 *a*) has been solved in the case $\Gamma_o = 0$. This corresponds to the neglect of any interaction between the substrates surrounding the sample and the smectic layers, which in this case can be assumed to be free. The validity of such an approximation of course depends on the choice of substrates, the surface treatment, etc. Taking account of Γ_o in equation (19 *a*), it is easily seen that the equilibrium orientation of the smectic layers is shifted from $\gamma = \beta_o$ (the smectic layer normal being parallel to the magnetic field) to a situation where the layer normal and the magnetic field are at a finite angle to each other. This is because the driving torque acting on the layer normal decreases with decreasing angle between the layer normal and the magnetic field. For a small enough angle, the stabilizing torque Γ_o is sufficiently large to balance the driving torque and the rotation of the smectic layers ceases. The angle between the layer normal and the magnetic field in this case is easily calculated from equation (19 *a*) as

$$\gamma_{eq} = \eta_o - \frac{1}{2}\arcsin\frac{2\Gamma_o}{\delta d}. \quad (35)$$

Table. Values of parameters used in this work.

α [2]	χ_a [18]	$\beta_2 \equiv \gamma_s$ [17]	τ_m [this work]	τ_l [3, 4, 14]	$\beta_1 - \beta_{12}$ [this work]
5×10^4 N m^{-2} K^{-1}	10^{-5} (SI)	1 Pa s	10^{-5} s	10 s	2000 Pa s

This equation is of course only valid if the value of Γ_o is so small that layer rotation is not prevented also for $\gamma = 0$. It is obvious that to achieve a system in the bookshelf geometry, some kind of surface treatment must be imposed on the glass plates surrounding the liquid crystal. Due to this, there must exist a stabilizing torque Γ_s, of maximum value Γ_o, which is responsible for keeping the preferred orientation of the layers. However, no systematic experimental studies of the interaction between the smectic layers and the substrates exist today. As is seen from equation (35), a quantitative measure of this interaction is given by Γ_o, which can determined experimentally with the aid of this equation. In the case where the angle $\eta_o - \gamma_{eq}$ is small, corresponding to the condition $2\Gamma_o \ll \delta d$, equation (35) can be approximated by—δ being given by equation (10)

$$\eta_o - \gamma_{eq} \approx \frac{\Gamma_o}{\delta d} = \frac{2\mu_o \Gamma_o}{\chi_a B^2 d} \tag{36}$$

and thus, in this limit, Γ_o can easily be evaluated from experiments by plotting $\eta_o - \gamma_{eq}$ as a function of $1/B^2$.

The parameters governing the behaviour of the system are, apart from the two control parameters B and $T - T_c$, the counter torque Γ_o discussed above and a set of material parameters for the liquid crystal studied. These parameters are the magnetic anisotropy χ_a, the Landau coefficient α and the three viscosities β_1, β_2, and β_{12}. Of these three viscosities β_2 represents the soft mode rotational viscosity while β_1 and β_{12} are viscosity coefficients connected to layer rotations. Values for the three parameters χ_a [18], α [2] and $\beta_2 \equiv \gamma_s$ [17] can be found in the literature, but no measurements of β_1 and β_{12} exist today. However, as is seen from equations (32) and (33), as long as the system is not too close to T_c, it is rather the combination $\beta_1 - \beta_{12}$ which determines the response of the system. By assuming $\tau_l \sim 10$ s, it was shown in the last section—equation (34)—that one can estimate $\beta_1 - \beta_{12} \sim 2000$ Pa s. This is a very large value, but only reflects the fact that this coefficient is connected to the rotation of entire smectic layers, a rotation which indeed has been observed to be very slow.

To summarize, typical values for the material parameters entering the problem studied in this work are given in the table. The values of some of these are established in the literature, but the numerical values in the table can be regarded only as a rough guideline, because no specific compound is being considered. Concerning the value of $\beta_1 - \beta_{12}$, this is estimated by assuming the response time of the smectic layers to be of the order of 10 s. It is of great importance for future work to determine τ_l experimentally in order to establish a more reliable value of the coefficient $\beta_1 - \beta_{12}$, which is the coefficient regulating the rotational motion of the smectic layers in the present problem.

References

[1] Carlsson, T., Leslie, F. M., and Clark, N. A., 1995, *Phys. Rev. E*, **51,** 4509.

[2] Carlsson, T., and Dahl, I., 1983, *Mol. Cryst. liq. Cryst.*, **95,** 373.

[3] Andersson, G., Carlsson, T., Lagerwall, S. T., and Matuszczyk, M., 1993, presented at the 4th International Ferroelectric Liquid Crystal Conference, Tokyo, Japan.

[4] Nakayama, K., Moritake, H., Ozaki, M., and Yoshino, K., 1995, *Jpn. J. appl. Phys.*, **34,** L1599.

[5] Nakayama, K., Togo, T., Ozaki, M., and Yoshono, K., 1998, *Ferroelectrics*, **212,** 1995.

[6] Dierking, I., Komitov, L., and Lagerwall, S. T., 1998, *Liq. Cryst.*, **24,** 775.

[7] Nakayama, K., Ozaki, M., and Yoshino, K., 1997, *Appl. Phys. Lett.*, **70,** 2117.

[8] Nakayama, K., Ozaki, M., and Yoshino, K., 1996, *Jpn. J. appl. Phys.*, **35,** 6200.

[9] Zalar, B., Gregorovic, A., Simsic, M., Zidansek, A., and Blinc, R., 1998, *Phys. Rev. Lett.*, **80,** 4458.

[10] Carlsson, T., Zeks, B., Filipic, C., and Levstik, A., 1990, *Phys. Rev. A*, **42,** 877.

[11] Carlsson, T., and Osipov, M. A., 1999, *Phys. Rev. E*, **60,** 5619.

[12] Carlsson, T., and Osipov, M. A. (to be published).

[13] Vertogen, G., and de Jeu, W. H., 1988, *Thermotropic Liquid Crystals, Fundamentals*, Vol. 45 in Springer Series in Chemical Physics, pp. 140–147.

[14] Andersson, G., and Dierking, I., personal communication.

[15] de Jeu, W. H., 1980, *Physical Properties of Liquid Crystalline Materials* (New York: Gordon and Breach), p. 76.

[16] Carlsson, T., Zeks, B., Filipic, C., Levstik, A., and Blinc, R., 1988, *Mol. Cryst. liq. Cryst.*, **163,** 11.

[17] Gouda, F., Skarp, K., and Lagerwall, S. T., 1991, *Ferroelectrics*, **113,** 165.

[18] de Jeu, W. H., 1980, *Physical Properties of Liquid Crystalline Materials* (New York: Gordon and Breach), p. 25.